On attractor mechanism and entropy function for non-extremal black holes/branes

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ABSTRACT: We examine in details the entropy function formalism for non-extremal $D3$, $M2$, and $M5$-branes that their throat approximation is given by Schwarzschild black hole in $AdS_{p+2} \times S^{D-(p+2)}$. We show that even though there is no attractor mechanism in the non-extremal black holes/branes, the entropy function formalism does work and the entropy is given by the entropy function at its saddle point.

KEYWORDS: D-branes, Black Holes in String Theory.

Contents

1. Introduction

The black hole attractor mechanism has been an active subject over the past few years in string theory. This is originated from the observation that there is a connection between the partition function of four-dimensional BPS black holes and partition function of topological strings [1].

The attractor mechanism states that in the extremal black hole backgrounds the moduli scalar fields at horizon are determined by the charge of black hole and are independent of their asymptotic values. One may study the attractor mechanism by finding the effective potential for the moduli fields and examining the behavior of the effective potential at its extremum, i.e., in order to have the attractor mechanism, the effective potential must have minimum in all directions. The entropy of black hole is then given by the value of the effective potential at its minimum. Using this, the entropy of some extremal black holes has been calculated in [2].

Motivated by the attractor mechanism, it has been proposed by A. Sen that the entropy of a specific class of extremal black holes in higher derivative gravity can be calculated using the entropy function formalism [3]. According to this formalism, the entropy function for the black holes that their near horizon is $AdS_2 \times S^{D-2}$ is defined by integrating the Lagrangian density over S^{D-2} for a general $AdS_2 \times S^{D-2}$ background characterized by the size of AdS_2 and S^{D-2}, and taking the Legendre transform of the resulting function with respect to the parameters labeling the electric fields. The result is a function of moduli scalar fields as well as the size of AdS_2 and S^{D-2}. The values of moduli fields and the sizes are determined by extremizing the entropy function with respect to the moduli fields and the sizes. Moreover, the entropy is given by the value of the entropy function at the

extremum.[1] Using this method the entropy of some extremal black holes have been found in [3 – 5].

For non-extremal black holes, one expects to have no attractor mechanism. An intuitional explanation of attractor mechanism has been proposed in [7]. According to which the physical distance from an arbitrary point to the horizon is infinite for black holes which have attractive horizon. While the physical distance is infinite for extremal black holes, it is finite for non-extremal cases. Alternatively, it has been shown in [8] that the values of the moduli fields at the horizon of non-extremal black holes depend on the asymptotic values of the scalar fields, hence, one expects to have no attractor mechanism for the non-extremal cases.

It is natural to ask if the entropy function formalism works for a non-extremal black hole. We speculate that the entropy function formalism works if the background is some extension of AdS at its near horizon. Moreover, for this background the entropy function has saddle point at the near horizon. In general, non-extremal black hole/brane solutions can be classified into three classes: 1) Solutions with no moduli, 2) Solutions with constant moduli, 3) Solutions with constant moduli at the near horizon. In this paper, we would like to consider the non-extremal black-branes whose near horizons are Schwarzschild black hole in $AdS_{p+2} \times S^{D-(p+2)}$. For $p = 3$, the solution is the non-extremal $D3$-brane with constant moduli. For $p = 2, 5$, the solutions are the non-extremal $M2$ and $M5$-branes with no moduli. We will discuss also the non-extremal black hole solutions with constant moduli at the near horizon which has been considered in [9].

An outline of the paper is as follows. In section 2, we review the non-extremal solutions of IIB/M theory. In sections 3 to 5, using the entropy function formalism we derive the known results for the entropy of $D3$, $M2$ and $M5$-branes in terms of the temperature. We also show that in all cases the entropy is given by the entropy function at its saddle point. In section 6 we show that the higher derivative terms do not respect the symmetries of the solution at tree level and so the entropy function formalism does not work. Instead, we use the Wald formula directly to find the correction to the entropy. We conclude with a discussion of our results in the last section.

2. Review of the non-extremal solutions

In this section, we review the non-extremal solutions of IIB/M theory. The two-derivative effective action for IIB/M theory in Einstein frame is given by

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left\{ R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \sum \frac{1}{n!} F_{(n)}^2 + \cdots \right\}, \qquad (2.1)$$

where $D = 10$ for IIB and $D = 11$ for M theory. In above Lagrangian ϕ is the dilaton which appears only in IIB theory, and $F_{(n)}$ is the electric field strength where $n = 1, 3, 5$ for

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[1] It is assumed that in the presence of higher derivative terms there is a solution whose near horizon geometry is $AdS_2 \times S^{D-2}$. In the cases that the higher derivative corrections modify the solution such that the near horizon is not $AdS_2 \times S^{D-2}$ anymore, one cannot use the entropy function formalism. In those cases one may use the Wald formula [6] to calculated the entropy directly.

IIB theory and $n = 4, 7$ for M theory. The $n = 5$ field strength tensor is self-dual, hence, it is not described by the above simple action. It is sufficient to adopt the above action for deriving the equations of motion, and impose the self-duality by hand. Dots represent fermionic terms as well as NS-NS 3-form field strength for IIB theory.

We are interested in non-extremal solutions whose near horizon are product space of AdS with a sphere. $D3$, $M2$ and $M5$-branes have this property. These solutions are given by the following (see e.g. [10]):

$$ds^2 = H^{-\frac{d-2}{D-2}}\left(-f dt^2 + \sum_{i=1}^{p}(dx^i)^2 \right) + H^{\frac{p+1}{D-2}}\left(f^{-1}dr^2 + r^2(d\Omega_{d-1})^2 \right),$$

$$e^\phi = 1 , \qquad F_{ti_1\cdots i_p r} = \epsilon_{i_1\cdots i_p} H^{-2}\frac{Q}{r^{d-1}} ,$$

$$H = 1 + \left(\frac{h}{r}\right)^{d-2} , \qquad f = 1 - \left(\frac{r_0}{r}\right)^{d-2} , \tag{2.2}$$

where $D = (p+1) + d$ and d is the number of dimensions transverse to the p-brane. Note that for $p = 3$, the above field strength is only the electric part of the self-dual $F_{(5)}$. We will see shortly that in the entropy function formalism one needs to consider only this part of $F_{(5)}$. The relation between h and Q is

$$h^{2(d-2)} + h^{d-2}r_0^{d-2} = \frac{Q^2}{(d-2)^2} . \tag{2.3}$$

For $r_0 = 0$ we obtain the extremal solution, depending only on a single parameter, Q, related to the common mass and charge density of the BPS p-branes.

For $r_0 \neq 0$ a horizon develops at $r = r_0$. The near horizon geometry which is described by a throat can be found by using the throat approximation where $r \ll h$. In this limit the relation (2.3) simplifies to $h^{d-2} = Q/(d-2)$, and the non-extremal solution becomes

$$ds^2 = \left(\frac{r}{h}\right)^{\frac{2(d-2)}{p+1}}\left\{ -\left[1-\left(\frac{r_0}{r}\right)^{d-2}\right]dt^2 + \sum_{i=1}^{p}(dx^i)^2 \right\} + \left(\frac{h}{r}\right)^2\left[1-\left(\frac{r_0}{r}\right)^{d-2}\right]^{-1}dr^2$$

$$+ h^2(d\Omega_{d-1})^2 ,$$

$$e^\phi = 1 , \qquad F_{ti_1\cdots i_p r} = (d-2)\epsilon_{i_1\cdots i_p}\frac{r^{d-3}}{h^{d-2}} , \tag{2.4}$$

where the geometry is the product of S^{d-1} with the Schwarzschild black hole in AdS_{D-d+1}.

3. Entropy function for non-extremal $D3$-branes

Following [3], in order to find the entropy function for non-extremal $D3$-branes one can deform the near horizon geometry as

$$ds_{10}^2 = v_1\left[\frac{r^2}{h^2}\left\{ -\left(1-\left(\frac{r_0}{r}\right)^4\right)dt^2 + \sum_{i=1}^{3}(dx^i)^2 \right\} + \frac{h^2}{r^2}\left(1-\left(\frac{r_0}{r}\right)^4\right)^{-1}dr^2 \right] + v_2 h^2(d\Omega_5)^2,$$

$$e^\phi = 1 , \qquad F_{ti_1i_2i_3 r} = 4\epsilon_{i_1i_2i_3}\left(\frac{v_1}{v_2}\right)^{5/2}\frac{r^3}{h^4} \equiv \epsilon_{i_1i_2i_3}e_1 , \tag{3.1}$$

where v_1 and v_2 are supposed to be constants. Note that we have considered only the electric part of the self-dual $F_{(5)}$. The function f is define to be the integral of Lagrangian density over the horizon $H = S^3 \times S^5$. The result of inserting the background of (3.1) into f is

$$
\begin{aligned}
f(v_1, v_2, e_1) &\equiv \frac{1}{16\pi G_{10}} \int dx^H \sqrt{-g} \mathcal{L} \\
&= \frac{V_3 V_5 h^2 r^3}{16\pi G_{10}} v_1^{5/2} v_2^{5/2} \left(\frac{20(v_1 - v_2)}{v_1 v_2 h^2} + \frac{h^6}{2v_1^5 r^6} e_1^2 \right),
\end{aligned}
\tag{3.2}
$$

where V_3 and V_5 are the volumes of 3 and 5-sphere with radius one. The electric charge carries by the brane is given by

$$
q_1 = \frac{\partial f}{\partial e_1} = \frac{V_3 V_5}{16\pi G_{10}} Q.
\tag{3.3}
$$

Now we define the entropy function by taking the Legendre transform of the above integral with respect to electric field e_1, that is

$$
\begin{aligned}
F(v_1, v_2, q_1) &\equiv e_1 \frac{\partial f}{\partial e_1} - f \\
&= \frac{V_3 V_5 h^2 r^3}{16\pi G_{10}} v_1^{5/2} v_2^{5/2} \left(-\frac{20(v_1 - v_2)}{v_1 v_2 h^2} + \frac{h^6}{2v_1^5 r^6} e_1^2 \right).
\end{aligned}
\tag{3.4}
$$

Substituting the value of e_1 and solving the equations of motion

$$
\frac{\partial F}{\partial v_i} = 0, \quad i = 1, 2,
\tag{3.5}
$$

one finds the following solution

$$
v_1 = 1, \quad v_2 = 1.
\tag{3.6}
$$

Let us now consider the behavior of the entropy function around the above critical point. To this end consider the following matrix

$$
M_{ij} = \partial_{v_i} \partial_{v_j} F(v_1, v_2).
\tag{3.7}
$$

Ignoring the overall constant factor, the eigenvalues of this matrix are $10(5 \pm \sqrt{89})$. This shows that the critical point $v_1 = v_2 = 1$ is a saddle point of the entropy function.

Let us now return to the entropy associated with this solution. It is straightforward to find the entropy from the Wald formula [6]

$$
S_{BH} = -\frac{8\pi}{16\pi G_{10}} \int dx^H \sqrt{g^H} \frac{\partial \mathcal{L}}{\partial R_{trtr}} g_{tt} g_{rr}.
\tag{3.8}
$$

For this background we have $R_{trtr} = \frac{r^4 - 3r_0^4}{v_1 h^2 r^4} g_{tt} g_{rr}$ and $\sqrt{-g} = v_1 \sqrt{g^H}$. These simplify the entropy relation to

$$
S_{BH} = -\frac{8\pi h^2 r^4}{16\pi G_{10}(r^4 - 3r_0^4)} \int dx^H \sqrt{-g} \frac{\partial \mathcal{L}}{\partial R_{trtr}} R_{trtr} = -\frac{2\pi h^2 r^4}{r^4 - 3r_0^4} \frac{\partial f_\lambda}{\partial \lambda} \bigg|_{\lambda=1},
\tag{3.9}
$$

where $f_\lambda(v_1, v_2, e_1)$ is an expression similar to $f(v_1, v_2, e_1)$ except that each R_{trtr} Riemann tensor component is scaled by a factor of λ.

To find $\frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1}$ using the prescription given in [3] and [4], we note that in addition to R_{trtr} the other Riemann tensor components $R_{ti_1ti_1}$, $R_{ri_1ri_1}$, and $R_{i_1i_2i_1i_2}$ where $i_1, i_2 = 1, 2, 3$ are all proportional to v_1, i.e.,

$$
R_{trtr} = v_1 \frac{3r_0^4 - r^4}{h^2 r^4}, \qquad R_{ri_1ri_1} = v_1 \frac{r^4 + r_0^4}{h^2(r^4 - r_0^4)},
$$
$$
R_{ti_1ti_1} = v_1 \frac{r_0^8 - r^8}{h^6 r^4}, \qquad R_{i_1i_2i_1i_2} = v_1 \frac{r^4 - r_0^4}{h^6}. \tag{3.10}
$$

Hence, one should rescale them too. We use the following scaling for these components

$$
R_{ti_1ti_1} \to \lambda_1 R_{ti_1ti_1}, \quad R_{ri_1ri_1} \to \lambda_2 R_{ri_1ri_1}, \quad R_{i_1i_2i_1i_2} \to \lambda_3 R_{i_1i_2i_1i_2}. \tag{3.11}
$$

Now we see that $f_\lambda(v_1, v_2, e_1)$ must be of the form $v_1^{5/2} g(v_2, \lambda v_1, e_1 v_1^{-5/2}, \lambda_1 v_1, \lambda_2 v_1, \lambda_3 v_1)$ for some function g. Then one can show that the following relation holds for f_λ and its derivatives with respect to scales, λ_i, e_1 and v_1:

$$
\lambda \frac{\partial f_\lambda}{\partial \lambda} + 3\lambda_1 \frac{\partial f_\lambda}{\partial \lambda_1} + 3\lambda_2 \frac{\partial f_\lambda}{\partial \lambda_2} + 3\lambda_3 \frac{\partial f_\lambda}{\partial \lambda_3} + \frac{5}{2} e_1 \frac{\partial f_\lambda}{\partial e_1} + v_1 \frac{\partial f_\lambda}{\partial v_1} - \frac{5}{2} f_\lambda = 0. \tag{3.12}
$$

In addition, there is another relation between the rescaled Riemann tensor components at the supergravity level which can be found using (3.10)

$$
3 \frac{\partial f_\lambda}{\partial \lambda_1}|_{\lambda_1=1} + 3 \frac{\partial f_\lambda}{\partial \lambda_2}|_{\lambda_2=1} + 3 \frac{\partial f_\lambda}{\partial \lambda_3}|_{\lambda_3=1} = \frac{3(3r^4 + r_0^4)}{r^4 - 3r_0^4} \frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1}. \tag{3.13}
$$

Replacing the above relation into (3.12) and using the equations of motion, one finds that $\frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1} = -\frac{1}{4} \frac{r^4 - 3r_0^4}{r^4} F$. It is easy to see that the entropy is proportional to the entropy function up to a constant coefficient, i.e.,

$$
S_{BH} = \frac{\pi h^2}{2} F = \frac{V_3 V_5 h^2 r_0^3}{4 G_{10}}. \tag{3.14}
$$

One may write the entropy in terms of temperature. The relation between r_0 and temperature can be read from the metric which is $r_0 = \pi h^2 T$, so

$$
S_{BH} = \frac{\pi^2}{2} N^2 V_3 T^3, \tag{3.15}
$$

where we have used the relations $V_5 = \pi^3$, $h^4 = \frac{N\kappa_{10}}{2\pi^{5/2}}$, and $2\kappa_{10}^2 = 16\pi G_{10}$ where N is the number of D3-branes. This is the entropy that has been found in [11]. Note that for extremal case, $r_0 = 0$, the entropy function is exactly the same as non-extremal case however, the value of entropy is zero.

We have seen that the entropy function formalism works very well here despite the fact that the horizon is not attractive. To see the latter fact, we note that the only scalar field in this theory is constant everywhere, and it does not appear in the Lagrangian. Therefore,

it is better to check the attractor property by calculation of the proper distance of an arbitrary point from the horizon, i.e.,

$$\rho = \int_{r_0}^{r} \frac{h}{r} \left(1 - \frac{r_0^4}{r^4}\right)^{-\frac{1}{2}} dr = \frac{1}{2} h \log \left[\frac{r^2}{r_0^2} + \sqrt{\frac{r^4}{r_0^4} - 1}\right], \qquad (3.16)$$

the above value is finite for non-extremal case but it is infinite for extremal case i.e., $r_0 \to 0$. Hence, although the attractor mechanism does not work for this non-extremal case, the entropy function formalism works and it gives the correct value of the entropy as the saddle point of the entropy function.

4. Entropy function for non-extremal $M2$-branes

The near horizon geometry of non-extremal $M2$-branes is described by the Schwarzschild $AdS_4 \times S^7$. The most general solution consistent with the symmetry of $AdS_4 \times S^7$ is

$$ds^2 = v_1 \left[\frac{y^2}{h^2} \left\{-\left(1 - \left(\frac{y_0}{y}\right)^3\right) dt^2 + \sum_{i=1}^{2}(dx^i)^2\right\} + \frac{h^2}{4y^2}\left(1 - \left(\frac{y_0}{y}\right)^3\right)^{-1} dy^2\right] + v_2 h^2 (d\Omega_7)^2,$$

$$F_{ti_1 i_2 y} = 3\epsilon_{i_1 i_2} \frac{v_1^2}{v_2^{7/2}} \frac{y^2}{h^3} \equiv \epsilon_{i_1 i_2} e_1, \qquad (4.1)$$

where we have defined the new variable $y = r^2/h$. In above v_1 and v_2 are constants. The value of entropy function in this case is given by

$$F = \frac{V_2 V_7 h^5 y^2}{32\pi G_{11}} v_1^2 v_2^{7/2} \left(-\frac{42v_1 - 48v_2}{v_1 v_2 h^2} + \frac{2h^4}{v_1^4 y^4} e_1^2\right), \qquad (4.2)$$

where V_2 and V_7 are the volume of 2 and 7-sphere with radius one. Substituting the value of e_1 and then solving the equations of motion gives $v_1 = v_2 = 1$. Moreover, the eigenvalues of the matrix (3.7) in this case are $3(83 \pm \sqrt{12937})$. So this shows again that the critical point $v_1 = v_2 = 1$ is the saddle point of the entropy function.

The entropy associated with this background is given by the Wald formula

$$S_{BH} = -\frac{8\pi}{16\pi G_{11}} \int dx^H \sqrt{g^H} \frac{\partial \mathcal{L}}{\partial R_{tyty}} g_{tt} g_{yy}. \qquad (4.3)$$

For the background (4.1) we find $R_{tyty} = \frac{4(y^3 - y_0^3)}{v_1 h^2 y^3} g_{tt} g_{yy}$ and $\sqrt{-g} = \frac{1}{2} v_1 \sqrt{g^H}$ so that the entropy can be written as

$$S_{BH} = -\frac{4\pi h^2 y^3}{16\pi G_{11}(y^3 - y_0^3)} \int dx^H \sqrt{-g} \frac{\partial \mathcal{L}}{\partial R_{tyty}} R_{tyty} = -\frac{\pi h^2 y^3}{y^3 - y_0^3} \frac{\partial f_\lambda}{\partial \lambda}\bigg|_{\lambda=1}. \qquad (4.4)$$

where again we have rescaled every factor R_{tyty} by λ in f_λ. In addition to R_{tyty}, there are three other types of Riemann curvature tensors which are proportional to v_1. These are $R_{ti_1 ti_1}$, $R_{yi_1 yi_1}$ and $R_{i_1 i_2 i_1 i_2}$ with $i_1, i_2 = 1, 2$, i.e.,

$$R_{tyty} = v_1 \frac{y_0^3 - y^3}{h^2 y^3}, \qquad\qquad R_{yi_1 yi_1} = v_1 \frac{2y^3 + y_0^3}{h^2(2y^3 - 2y_0^3)},$$

$$R_{ti_1 ti_1} = -v_1 \frac{4y^6 - 2y^3 y_0^3 - 2y_0^6}{h^6 y^2}, \qquad R_{i_1 i_2 i_1 i_2} = v_1 \frac{4y^4 - 4yy_0^3}{h^6}. \qquad (4.5)$$

Rescaling them with λ_1, λ_2 and λ_3 as in (3.11) and noting that $f_\lambda(v_1, v_2, e_1)$ must be of the form $v_1^2 g(v_2, \lambda v_1, e_1 v_1^{-2}, \lambda_1 v_1, \lambda_2 v_1, \lambda_3 v_1)$ for some function g, one finds the following relation:

$$\lambda \frac{\partial f_\lambda}{\partial \lambda} + 2\lambda_1 \frac{\partial f_\lambda}{\partial \lambda_1} + 2\lambda_2 \frac{\partial f_\lambda}{\partial \lambda_2} + \lambda_3 \frac{\partial f_\lambda}{\partial \lambda_3} + 2e_1 \frac{\partial f_\lambda}{\partial e_1} + v_1 \frac{\partial f_\lambda}{\partial v_1} - 2f_\lambda = 0 \,, \qquad (4.6)$$

using (4.5), one finds

$$2 \frac{\partial f_\lambda}{\partial \lambda_1}\bigg|_{\lambda_1=1} + 2\frac{\partial f_\lambda}{\partial \lambda_2}\bigg|_{\lambda_2=1} + \frac{\partial f_\lambda}{\partial \lambda_3}\bigg|_{\lambda_3=1} = \frac{5y^3 + y_0^3}{y^3 - y_0^3} \frac{\partial f_\lambda}{\partial \lambda}\bigg|_{\lambda=1} \,. \qquad (4.7)$$

Replacing the above relation into the (4.6), one finds $\frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1} = -\frac{y^3-y_0^3}{3y^3}F$ and therefore

$$S_{BH} = \frac{\pi h^2}{3} F = \frac{V_2 V_7 h^5 y_0^2}{4G_{11}} \,, \qquad (4.8)$$

this gives a non-zero value for entropy. One may write the entropy in terms of temperature. From the metric (4.1) we find the relation between non-extremality parameter and temperature as $y_0 = 2\pi h^2 T/3$. So the entropy becomes

$$S_{BH} = 2^{7/2} 3^{-3} \pi^2 V_2 N^{3/2} T^2 \qquad (4.9)$$

where we have used the relations $V_7 = \pi^4/3$, $h^9 = N^{3/2} \frac{\kappa_{11}^2 \sqrt{2}}{\pi^5}$ and $2\kappa_{11}^2 = 16\pi G_{11}$ where N is the number of M2-branes. This is the entropy, which has been found in [11]. Note again that the extremal case can be found by taking $y_0 = 0$. The result for entropy function is exactly the same as non-extremal case but the value of entropy is zero.

To check the attractor mechanism, we note that there is no scalar field in this theory so we calculate the proper distance of an arbitrary point from the horizon, i.e.,

$$\rho = \int_{y_0}^y \frac{h}{2y} \left(1 - \frac{y_0^3}{y^3}\right)^{-\frac{1}{2}} dy = \frac{1}{3} h \log\left[\left(\frac{y}{y_0}\right)^{\frac{3}{2}} + \sqrt{\left(\frac{y}{y_0}\right)^3 - 1}\right] \,, \qquad (4.10)$$

which is finite for the non-extremal case but is infinite when $y_0 \to 0$ in the extremal case. This shows again that although the horizon is not attractive point, the entropy function formalism works and it gives the correct value for the entropy as the saddle point of the entropy function.

5. Entropy function of non-extremal $M5$-branes

For non-extremal M5-branes the background is Schwarzschild $AdS_7 \times S^4$ and the general solution consistent with this symmetry is

$$ds_{11}^2 = v_1 \left[\frac{y^2}{h^2} \left\{ -\left(1 - \left(\frac{y_0}{y}\right)^6\right) dt^2 + \sum_{i=1}^5 (dx^i)^2 \right\} + 4\frac{h^2}{y^2} \left(1 - \left(\frac{y_0}{y}\right)^6\right)^{-1} dy^2 \right] + v_2 h^2 (d\Omega_4)^2 \,,$$

$$F_{ti_1\cdots i_5 r} = 6\epsilon_{i_1\cdots i_5} \frac{v_1^{7/2}}{v_2^2} \frac{y^5}{h^6} \equiv \epsilon_{i_1\cdots i_5} e_1 \,, \qquad (5.1)$$

where we have used the new coordinate $y = \sqrt{hr}$. The entropy function for this background is

$$F = \frac{2V_5 V_4 y^5}{16\pi G_{11} h} v_1^{7/2} v_2^2 \left(-\frac{24v_1 - 21v_2}{2v_1 v_2 h^2} + \frac{h^{10}}{8v_1^7 y^{10}} e_1^2 \right), \tag{5.2}$$

where V_5 and V_4 are the volume of 5 and 4-sphere with radius one. Substituting the value of e_1 and solving the equations of motion results $v_1 = v_2 = 1$. The eigenvalues of the matrix (3.7) in this case are $\frac{3}{8}(29 \pm \sqrt{12937})$. Therefore it shows that the critical point $v_1 = v_2 = 1$ is a saddle point of the entropy function.

Let us now turn to the entropy associated with this solution. The Wald formula in (4.3) still holds here. Using the fact that for this background $R_{tyty} = \frac{y^6 - 10y_0^6}{4v_1 h^2 y^6} g_{tt} g_{yy}$ and $\sqrt{-g} = 2v_1 \sqrt{g^H}$ one finds

$$S_{BH} = -\frac{16\pi h^2 y^6}{16\pi G_{11}(y^6 - 10y_0^6)} \int dx^H \sqrt{-g} \frac{\partial \mathcal{L}}{\partial R_{tyty}} R_{tyty} = -\frac{4\pi h^2 y^6}{y^6 - 10y_0^6} \frac{\partial f_\lambda}{\partial \lambda}\bigg|_{\lambda=1}, \tag{5.3}$$

where we have rescaled R_{tyty} in f_λ. There are other Riemann tensor components proportional to v_1. These are $R_{ti_1 ti_1}$, $R_{yi_1 yi_1}$ and $R_{i_1 i_2 i_1 i_2}$ with $i_1, i_2 = 1 \ldots 5$, i.e.,

$$R_{tyty} = v_1 \frac{10y_0^6 - y^6}{h^2 y^6}, \qquad\qquad R_{yi_1 yi_1} = v_1 \frac{y^6 + 2y_0^6}{h^2(y^6 - y_0^6)},$$
$$R_{ti_1 ti_1} = -v_1 \frac{y^{12} + y^6 y_0^6 - 2y_0^{12}}{4h^6 y^8}, \qquad R_{i_1 i_2 i_1 i_2} = v_1 \frac{y^6 - y_0^6}{4h^6 y^2}. \tag{5.4}$$

We rescale them by λ_1, λ_2 and λ_3. Nothing that $f_\lambda(v_1, v_2, e_1)$ must be of the general form $v_1^{7/2} g(v_2, \lambda v_1, e_1 v_1^{-7/2}, \lambda_1 v_1, \lambda_2 v_1, \lambda_3 v_1)$ for some function g, one finds

$$\lambda \frac{\partial f_\lambda}{\partial \lambda} + 5\lambda_1 \frac{\partial f_\lambda}{\partial \lambda_1} + 5\lambda_2 \frac{\partial f_\lambda}{\partial \lambda_2} + 10\lambda_3 \frac{\partial f_\lambda}{\partial \lambda_3} + \frac{7}{2} e_1 \frac{\partial f_\lambda}{\partial e_1} + v_1 \frac{\partial f_\lambda}{\partial v_1} - \frac{7}{2} f_\lambda = 0, \tag{5.5}$$

One finds also the following relation at the supergravity level:

$$5\frac{\partial f_\lambda}{\partial \lambda_1}\bigg|_{\lambda_1=1} + 5\frac{\partial f_\lambda}{\partial \lambda_2}\bigg|_{\lambda_2=1} + 10\frac{\partial f_\lambda}{\partial \lambda_3}\bigg|_{\lambda_3=1} = 10\frac{2y^6 + y_0^6}{y^6 - 10y_0^6} \frac{\partial f_\lambda}{\partial \lambda}\bigg|_{\lambda=1}. \tag{5.6}$$

Replacing the above relation in (5.5) one can show that $\frac{\partial f_\lambda}{\partial \lambda}|_{\lambda=1} = -\frac{1}{6}\frac{y^6 - 10y_0^6}{y^6} F$ and therefore

$$S_{BH} = \frac{2\pi h^2}{3} F = \frac{V_5 V_4 y_0^5}{4G_{11} h}, \tag{5.7}$$

this gives non-zero result. To write the entropy in terms of temperature we use $y_0 = \frac{4\pi h^2 T}{3}$ then

$$S_{BH} = 2^7 3^{-6} \pi^3 N^3 V_5 T^5, \tag{5.8}$$

where we have used the relations $V_4 = \frac{8\pi^2}{3}$, $h^9 = \frac{N^3 \kappa_{11}^2}{2^7 \pi^5}$ and $2\kappa_{11}^2 = 16\pi G_{11}$ where N is the number of M5-branes. This is in agreement with the result in [11].

We look now to the attractor mechanism. As we see, there is no scalar field in this case so we check the attractor property by calculation of the proper distance of an arbitrary point from the horizon

$$\rho = \int_{y_0}^{y} \frac{2h}{y} \left(1 - \frac{y_0^6}{y^6}\right)^{-\frac{1}{2}} dy = \frac{2}{3} h \log\left[\left(\frac{y}{y_0}\right)^3 + \sqrt{\left(\frac{y}{y_0}\right)^6 - 1}\right],\qquad (5.9)$$

which is finite for the non-extremal case but is infinite when $y_0 \to 0$ in the extremal case. This shows again that although the horizon is not attractive for the non-extremal case, the entropy function formalism works and it gives the correct value for the entropy as the saddle point of the entropy function.

6. Higher derivative terms for non-extremal $D3$-branes

In the previous sections, we have seen that the entropy function works at two derivatives level. It will be interesting to consider stringy effects and look at the entropy function mechanism again. To this end, we consider the higher derivative corrections coming from string theory. To next leading order the Lagrangian of IIB theory in Einstein frame is given by

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \sum \frac{1}{n!} F_{(n)}^2 + \gamma e^{-3\phi/2} W \right\}, \quad (6.1)$$

where $\gamma = \frac{1}{8} \zeta(3)(\alpha')^3$ and W can be written in terms of the Weyl tensors

$$W = C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C_h{}^{rsp} C^q{}_{rsk}. \qquad (6.2)$$

In what follows, we will show that (3.1) is no longer a solution of the above action. We calculate the contribution of the above higher derivative terms to the entropy function F

$$\delta F = -\frac{\gamma}{16\pi G_{10}} \int dx^H \sqrt{-g} W =$$
$$= -\gamma \frac{V_3 V_5}{16\pi G_{10}} \frac{180}{h^6 r^{13} (v_1 v_2)^{3/2}} \left[\frac{35}{1944} (v_1 - v_2)^4 r^{16} + \frac{1}{6} v_2^2 (v_1 - v_2)^2 r_0^8 r^8 + v_2^4 r_0^{16} \right], (6.3)$$

By variation of $F + \delta F$ with respect to v_1 and v_2 one finds the equations of motion. Since these equations are valid only up to first order of γ, we consider the following perturbative solutions

$$v_1 = 1 + \gamma x, \quad v_2 = 1 + \gamma y. \qquad (6.4)$$

For extremal case, $r_0 = 0$, the corrections are zero, i.e., $v_1 = 1 = v_2$. Again the value of entropy is proportional to r^3 which gives zero. This is due to the fact that $AdS_5 \times S^5$ is an exact solution.

For non-extremal case, by replacing the above solutions into the equations of motion, one finds the following relations

$$\frac{\partial(F + \delta F)}{\partial v_1} = 0 \longrightarrow 3x + 5y = \frac{27}{h^6}\left(\frac{r_0}{r}\right)^{16},$$

$$\frac{\partial(F + \delta F)}{\partial v_2} = 0 \longrightarrow 5x - 13y = -\frac{45}{h^6}\left(\frac{r_0}{r}\right)^{16}, \qquad (6.5)$$

these equations are consistent, and give the following result

$$v_1 = 1 + \frac{63}{32h^6}\left(\frac{r_0}{r}\right)^{16}\gamma \ , \ v_2 = 1 + \frac{135}{32h^6}\left(\frac{r_0}{r}\right)^{16}\gamma \,. \qquad (6.6)$$

However, they are functions of r. This is inconsistent with our assumption that v_1 and v_2 are constants!. So it seems that the deformed geometry (3.1) is not the solution of equations of motion when we consider higher derivative terms. Hence the entropy function formalism does not work when higher derivative corrections are added to the effective action. The same thing happens for $M2$ and $M5$-branes.

This is related to the fact that in the presence of the higher derivative terms the solution is not the Schwarzschild AdS anymore. The ansatz for the metric should be [11]

$$ds^2 = r^2(-e^{2a+8b}dt^2 + e^{2b}dr^2 + d\vec{x}^2) + e^{2c}d\Omega_5^2 \,, \qquad (6.7)$$

where a, b and c are functions of r and we have chosen $h = 1$. The solution for these functions at linear order of γ gives a metric which is not the Schwarzschild AdS [11]. Using the ansatz (6.7), one realizes that the horizon area does not modify so the entropy is given by (3.9) where now f_λ is replaced by $f_\lambda + f_\lambda^W$, i.e.,

$$S_{BH} = -\frac{2\pi h^2 r^4}{r^4 - 3r_0^4}\frac{\partial(f_\lambda + f_\lambda^W)}{\partial\lambda}\bigg|_{\lambda=1}, \qquad (6.8)$$

where the function f^W is given by

$$f^W = \frac{\gamma}{16\pi G_{10}}\int dx^H\sqrt{-g}e^{-\frac{3}{2}\phi}W\,. \qquad (6.9)$$

The first term in (6.8) give the same result as before, i.e., (3.14). The second term is proportional to γ, so to the first order of γ one has to replace the Schwarzschild AdS solution (3.1) in $\frac{\partial f_\lambda^W}{\partial\lambda}$ which gives

$$\frac{\partial f_\lambda^W}{\partial\lambda}\bigg|_{\lambda=1} = -120\frac{V_3V_5}{16\pi G_{10}}\frac{(r^4 - 3r_0^4)r_0^{12}}{r^{13}}\,. \qquad (6.10)$$

Finally the entropy will be

$$S_{BH} = \frac{V_3V_5h^2}{4G_{10}}r_0^3\left(1 + 60\gamma + \mathcal{O}(\gamma^2)\right). \qquad (6.11)$$

In terms of temperature [11], $T = \frac{r_0}{\pi}(1 + 15\gamma)$, one finds

$$S_{BH} = \frac{\pi^2}{2}N^2V_3T^3\left(1 + 15\gamma\right). \qquad (6.12)$$

This is the entropy that has been found in [11] using the free energy formalism.

7. Discussion

In this paper, we have studied in details the entropy function formalism for non-extremal $D3$, $M2$ and $M5$-branes. We have shown that the entropy function can be applied to find the entropy of these solutions at tree level. The entropy function in all cases has a saddle point and the entropy is given by the value of this function at this point.

We have studied non-extremal black branes, which have either no moduli or constant moduli. The non-extremal black holes which have non-constant moduli, has been studied in [9]. One may expect that in this case also the entropy function should have a saddle point. To see this more explicitly let us consider the 5 dimensional non-extremal black holes in IIB theory compactified on $T^4 \times S^1$ with the following $BTZ \times S^2$ near horizon geometry [9]:

$$ds^2 = v_1 \left[-\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\rho^2} dt^2 + \frac{4\rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 \right.$$
$$\left. + \rho^2 \left(dz - \frac{\rho_+ \rho_-}{\rho^2} dt \right)^2 \right] + v_2 d\Omega_2^2,$$

$$e^{-2\phi} = u_s, \quad e^{2\psi} = u_T, \quad e^{\frac{\psi_1}{2}} = u_1,$$

$$F_{tz\rho}^{(5)} = e_1 = \frac{\rho u_1}{u_T} \frac{v_1^{\frac{3}{2}}}{v_2}, \quad H_{\theta\varphi}^{(5)} = -\frac{1}{2} \sin \theta, \quad G_{\theta\varphi} = -\frac{1}{2} \sin \theta, \tag{7.1}$$

where $e^{2\psi}$ and $e^{\frac{\psi_1}{2}}$ denote the single moduli for T^4 and S^1 respectively. We refer the reader to [9] for details. The entropy function in this case is proportional to

$$F \sim v_1^{\frac{3}{2}} v_2 u_T u_1 \left[u_s \left(\frac{3v_2 - 4v_1}{2v_1 v_2} + \frac{1}{2u_1^2 v_2^2} \right) + \frac{1}{2v_2^2} + \frac{e_1^2}{2u_1^2 \rho^2 v_1^3} \right]. \tag{7.2}$$

The solution to the equations of motion

$$\frac{\partial F}{\partial u_i} = 0, \quad i = s, T, 1, \qquad \frac{\partial F}{\partial v_j} = 0, \quad j = 1, 2, \tag{7.3}$$

is $v_1 = v_2 = v$, $u_s = \frac{1}{v}$, $u_T = 1$, $u_1 = \frac{1}{\sqrt{v}}$. As can be seen, these equations of motion cannot fix all the moduli so one expects that the entropy function has a flat direction [3]. To study the behavior of the entropy function around the above critical point, consider the following matrix:

$$M_{ij} = \partial_{\phi_i} \partial_{\phi_j} F, \quad \phi_i = \{v_1, v_2, u_s, u_T, u_1\}. \tag{7.4}$$

The eigenvalues of this matrix for $v = 1$ are

$$(4.81, -3.34, 2.23, 0.55, 0). \tag{7.5}$$

The negative eigenvalue indicates that the critical point is a saddle point. Moreover as anticipated above one of the eigenvalues is zero.

We have seen in sections 3, 4 and 5 that the entropy function has one minimum and one maximum in the directions specified by the sizes of AdS_{p+2} and $S^{D-(p+2)}$. This might

be related to the fact that curvature of AdS_{p+2} is negative and the curvature of $S^{D-(p+2)}$ is positive. This property should be independent of the attractiveness of the black holes. So one expects that this property holds even for extremal solutions with $AdS_{p+2} \times S^{D-(p+2)}$ near horizon. To see this, consider the Dyonic black holes in Heterotic string theory compactified on $M \times S^1 \times \tilde{S}^1$ where M is a four dimensional compact manifold and S^1 and $\tilde{S}^1$ are circles [3]. The near horizon geometry is given by

$$
\begin{aligned}
ds^2 &= v_1\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + v_2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \\
S &= u_S, \qquad R = u_R, \qquad \tilde{R} = u_{\tilde{R}}, \\
F_{rt}^{(1)} &= e_1, \quad F_{rt}^{(3)} = e_3, \quad F_{\theta\phi}^{(2)} = p_2, \quad F_{\theta\phi}^{(4)} = p_4,
\end{aligned}
\tag{7.6}
$$

where S is dilaton and R and $\tilde{R}$ are radii of the circles. The entropy function in terms of the electric and magnetic charges q_1, q_3, p_2, p_4 is given by

$$
F = \frac{\pi}{4} v_1 v_2 u_S \left[\frac{2}{v_1} - \frac{2}{v_2} + \frac{8q_1^2}{u_R^2 u_S^2 v_2^2} + \frac{8u_R^2 q_3^2}{u_S^2 v_2^2} + \frac{2u_{\tilde{R}}^2 p_2^2}{16\pi^2 v_2^2} + \frac{2p_4^2}{16\pi^2 u_{\tilde{R}}^2 v_2^2} \right] + 4\pi u_S,
\tag{7.7}
$$

where the charge quantization gives $q_1 = \frac{n}{2}, q_3 = \frac{w}{2}, p_2 = 4\pi\tilde{k}, p_4 = 4\pi\tilde{w}$. Solving equations of motion gives rise to the following solutions for scalars

$$
v_1 = v_2 = 4\tilde{n}\tilde{w} + 8, \quad u_S = \sqrt{\frac{nw}{\tilde{n}\tilde{w}+4}}, \quad u_R = \sqrt{\frac{n}{w}}, \quad u_{\tilde{R}} = \sqrt{\frac{\tilde{w}}{\tilde{n}}}.
\tag{7.8}
$$

We can construct the following matrix as before:

$$
M_{ij} = \partial_{\phi_i}\partial_{\phi_j} F, \quad \phi_i = \{v_1, v_2, u_S, u_R, u_{\tilde{R}}\},
\tag{7.9}
$$

For the case that $n = w = \tilde{n} = \tilde{w} = 1$ the eigenvalues are

$$
(70.44, \ 28.10, \ 5.62, \ -0.11, \ 0.03).
\tag{7.10}
$$

We see that as expected the critical point is a saddle point.

The eigenvalues (7.5) and (7.10) indicate that the critical point in both non-extremal and extremal solutions are the saddle points of the entropy function. However, the attractiveness of the solutions cannot be seen from these eigenvalues. The attractiveness can be studied either by the proper distance of an arbitrary point from the horizon [9] or by looking at the effective potential for the moduli fields. The effective potential can be read from the entropy function by inserting in the values of sizes v_1 and v_2. Doing this one finds that the eigenvalues of the matrix M_{ij} constructed from the effective potential, have negative values in non-extremal case whereas for extremal case, all the eigenvalues are positive.

The entropy function formalism works for those black holes/branes that their near horizon is an extension of AdS space. The near horizon (throat approximation) of the p-brane solutions ($D3$, $M2$, $M5$-branes) that we have studied are the Schwarzschild AdS times sphere. For other p-branes this near horizon is not a product space so the entropy

function formalism does not work. One may consider instead the near horizon (not the throat approximation) of the non-extremal p-brane solutions which is a product of the Rindler space times a sphere. It can easily be checked that the entropy function formalism does not work for this space.[2]

We have seen in the section 6 that the higher derivative corrections modify the tree level solutions such that the near horizon (throat approximation) is not the Schwarzschild AdS anymore. Consequently, the entropy function formalism does not work for these cases. Hence, we have used the Wald formula to find the value of entropy directly. It would be interesting to find a non-extremal solution where the higher derivative corrections respect the symmetries of the tree level solution i.e., AdS. In those cases, one would expect to find the entropy function including the higher derivative corrections by using the entropy function formalism [13].

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